De-jittering & Reconstruction of Images

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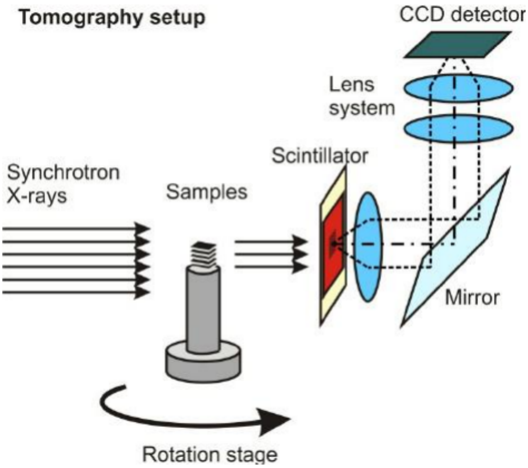
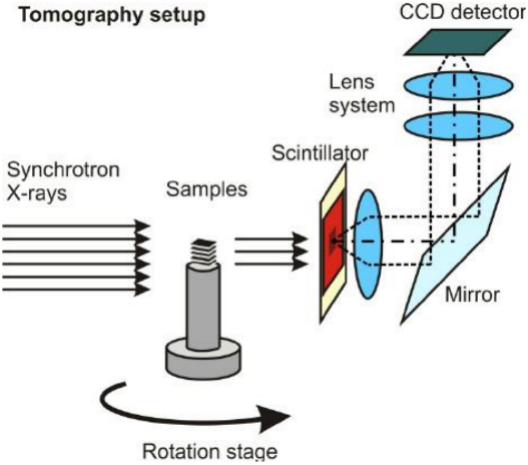
February 1, 2019

The Problem



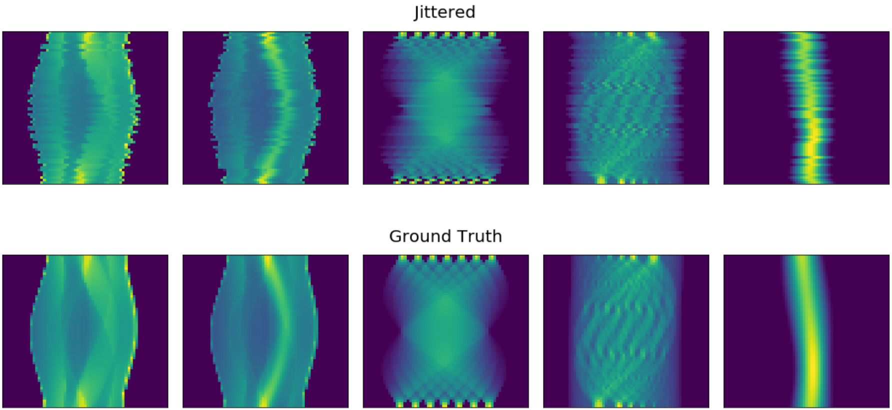
Tomography setup
CCD detector
Lens
system
Scintillator
Synchrotron
X-rays
Samples
Mirror
Rotation stage

The Problem



On a nano level, it is near impossible to place the sample back in it's exact original location

Toy Dataset



Ground truth available and jitter process available from tomophantom package from `https://github.com/dkazanc`

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- Define $TV(\cdot) := \sum_{i,j} |(\nabla \cdot)_{i,j}|$
- $\underline{\mathbf{s}}^* = \underset{\underline{\mathbf{s}}}{\operatorname{argmin}} \, TV(T_{\underline{\mathbf{s}}}(\mathbf{X}))$

Simple Toy Problem

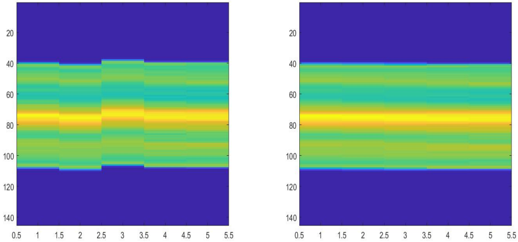
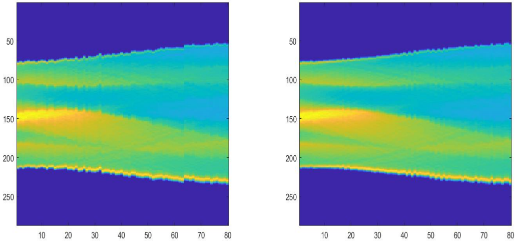


Figure: Before and after alignment

Shepp-Logan Phantom Sinogram Snippet



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Direct Image Reconstruction from Jittered Data

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$$g(\mathbf{U}) = \begin{cases} \infty, & \text{if any } u_{i,j} < 0 \\ \lambda TV(\mathbf{U}), & \text{otherwise} \end{cases}$$

Try to find the best slice of an image which has shifted by some unknown small value such that we see a particular column of the sinogram.

Direct Image Reconstruction from Jittered Data

Initialise $\underline{\mathbf{s}}$ randomly, $\epsilon > 0, \sigma_0 \in (0, 1]$

for $i = 1, ..., n\_iter$ **do**

$\mathbf{U}_i = \underset{\mathbf{U}}{\operatorname{argmin}} \sum_i (f_i(\mathbf{U})) + g(\mathbf{U})$

for $j = 1, ..., N$ **do**

for $k = 1, ..., K$ **do**

Draw $s_k \sim U(s_j - \epsilon, s_j + \epsilon)$

if $f_k(\mathbf{U}) < f_j(\mathbf{U})$ **then**

$s_j = s_k$

end

end

end

end

Square Toy Problem

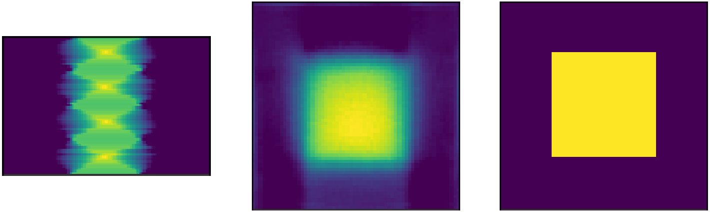
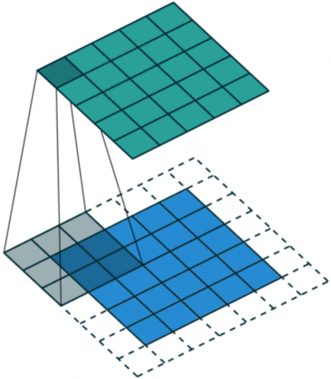
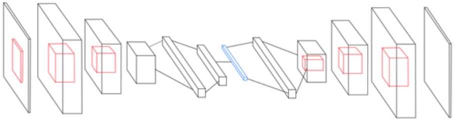


Figure: From left to right are the jittered sinogram, image reconstruction and ground truth

Deep Learning

Architecture: Convolutional Auto-Encoder

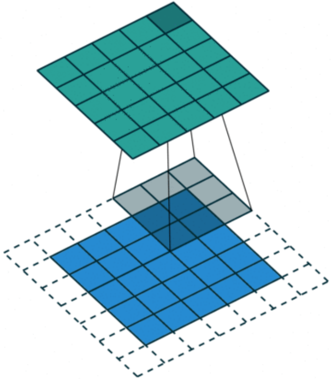


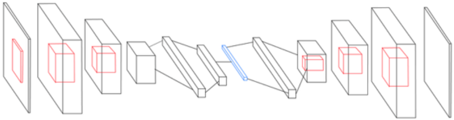


Deep Learning

Architecture: Convolutional Auto-Encoder

- Convolutions to extract local information

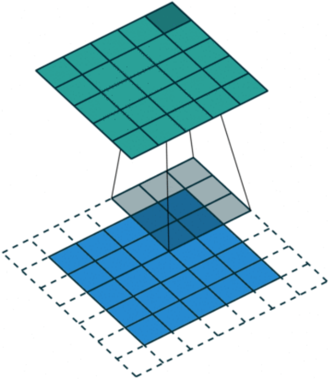


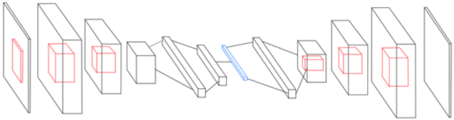


Deep Learning

Architecture: Convolutional Auto-Encoder

- Convolutions to extract local information
- Finds relationships between pixels in much lower dimensional space using data-driven non-linear PCA





Results